Technical Memorandum No. 33-163

Theoretical Noble-Gas Performances in an Ideal Constant-Area Shock Tube

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ABSTRACT

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The analytical procedures described were developed to support an experimental investigation of the thermal conductivity of noble gases in a constant-area shock tube. To provide a more efficient means of choosing initial pressures for the driver and driven gases in the tube, the gas dynamic equations were solved by an iterative method, and curves were plotted for ten combinations of helium or argon driving helium, neon, argon, krypton, or xenon. From these curves, the initial shock tube pressures may be easily determined for any given set of stagnation conditions.

Author

I. INTRODUCTION

The purpose of this Report is to present the results of a parametric study which was initiated to support the thermal-conductivity experiments being conducted in the 3-in.-D shock tube. In these experiments, the stagnation conditions existing behind the reflected shock wave were used to measure the thermal conductivity of various noble gases. For each set of stagnation conditions desired for the experimental program, it was necessary to solve the gas dynamic equations by an iterative method to obtain the required initial driver-gas and driven-gas pressures in the shock tube.

During the early stages of the thermal-conductivity experiments, it became obvious that a more efficient method of choosing the initial pressures was needed. To accomplish this, a digital computer was used to solve the ideal-gas dynamic equations, from which curves of driver pressure versus driven pressure were prepared with stagnation pressure and temperature as parameters. The shock tube gases used for the graphs presented here were combinations of helium or argon driving helium, neon, argon, krypton, or xenon.

II. ANALYTICAL PROCEDURE

A. Shock Tube Equations

The ratio of specific heats for the gases considered here is $\gamma = 5/3$. The applicable ideal-gas dynamic equations are as follows:

From Ref. 2, p. 59,

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma_1}{\gamma_1 + 1}(M_s^2 - 1) = 1.250\, M_s^2 - 0.250 \tag{1}$$

From Ref. 1, p. 83,

$$\frac{P_5}{P_2} = \frac{\alpha_1 + 2 - \dfrac{P_1}{P_2}}{1 + \alpha_1 \dfrac{P_1}{P_2}} = \frac{6 - \dfrac{P_1}{P_2}}{1 + 4\dfrac{P_1}{P_2}} \tag{2}$$

$$\frac{P_5}{P_1} = \frac{P_5}{P_2}\frac{P_2}{P_1} \tag{3}$$

From Ref. 2, p. 64,

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\alpha_1 + \dfrac{P_2}{P_1}}{1 + \alpha_1\dfrac{P_2}{P_1}} = \frac{4\dfrac{P_2}{P_1} + \left(\dfrac{P_2}{P_1}\right)^2}{1 + 4\dfrac{P_2}{P_1}} \tag{4}$$

From Ref. 1, p. 83,

$$\frac{T_5}{T_2} = \frac{\dfrac{P_5}{P_2}\left(\alpha_1 + \dfrac{P_5}{P_2}\right)}{1 + \alpha_1\dfrac{P_5}{P_2}} = \frac{4\dfrac{P_5}{P_2} + \left(\dfrac{P_5}{P_2}\right)^2}{1 + 4\dfrac{P_5}{P_2}} \tag{5}$$

$$\frac{T_5}{T_1} = \frac{T_5}{T_2}\frac{T_2}{T_1} \tag{6}$$

$$P_1 = \frac{P_1}{P_5}P_5 \tag{7}$$

And, from Ref. 1, p. 69,

$$\frac{P_4}{P_1} = \frac{1}{\alpha_1}\left(\frac{M_s^2}{\beta_1} - 1\right)\left[1 - \frac{\gamma_4 - 1}{\gamma_1 + 1}\frac{a_1}{a_4}\left(M_s - \frac{1}{M_s}\right)\right]^{-1/\beta_4}$$

$$\frac{P_4}{P_1} = 0.25\ (5M_s^2 - 1)\left[1 - 0.25\frac{a_1}{a_4}\left(M_s - \frac{1}{M_s}\right)\right]^{-5} \tag{8}$$

where

$$\frac{a_1}{a_4} = \left(\frac{\gamma_1}{\gamma_4} \frac{M_4}{M_1} \frac{T_1}{T_4} \right)^{1/2} \tag{9}$$

Also,

$$P_4 = \frac{P_4}{P_1} P_1 \tag{10}$$

B. *Method of Computation*

The computer procedures employed in this study are outlined below:

1. Equations 1 to 6 were solved for a shock Mach number range of 1.10 to 7.00. These solutions are plotted in Figs. 1 and 2.
2. Using Eq. 7, the initial driven pressure P_1 was calculated for each Mach number by setting the reflected pressure P_5 equal to 1, 2, and 5 atm.
3. For neon as the driven gas and helium as the driver gas, the shock tube pressure ratio P_4/P_1 was calculated from Eq. 8 for each Mach number.
4. Applying the results of steps 2 and 3 to Eq. 10 for each Mach number, the driver pressure P_4 was calculated for $P_5 = 1$, 2, and 5 atm.
5. Steps 3 and 4 were repeated for argon driving neon.
6. Steps 3, 4, and 5 were repeated for each of the other driven gases.

III. RESULTS

The final results of the computer calculations are shown in the working curves of Figs. 3 to 12. The stagnation conditions are presented in the form of T_5 isothermal lines and P_5 isobaric lines, while P_1 and P_4 are shown in units consistent with most shock tube measuring apparatus. From these curves, the initial shock tube pressures are easily found for any desired set of stagnation conditions.

It should be noted that the parametric curves presented here are for the ideal shock tube, where side-wall boundary-layer growth and shock wave attenuation are neglected. The application of these curves to a real shock tube in predicting the thermodynamic conditions obtainable from given initial pressures is inherently limited, then, to a first approximation. In the reduction of shock tube data where more accurate values of T_5 and P_5 are required, the measured shock speed, attenuated to the end-wall, should be used in Eqs. 2 and 4.

NOMENCLATURE

a speed of sound

m molecular weight

He : $m = 4.003$

Ne : $m = 20.183$

A : $m = 39.948$

Kr : $m = 83.80$

Xe : $m = 131.30$

M_s shock Mach number

P pressure

T temperature

α $\dfrac{\gamma + 1}{\gamma - 1}$

β $\dfrac{\gamma - 1}{2\gamma}$

γ specific heat ratio $= 5/3$

Subscripts

$_1$ initial driven-gas condition

$_2$ condition behind incident shock wave

$_4$ initial driver-gas condition

$_5$ condition behind reflected shock wave

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2. Liepmann, H. W., and Roshko, A., *Elements of Gasdynamics*, John Wiley & Sons, Inc., New York, 1957.

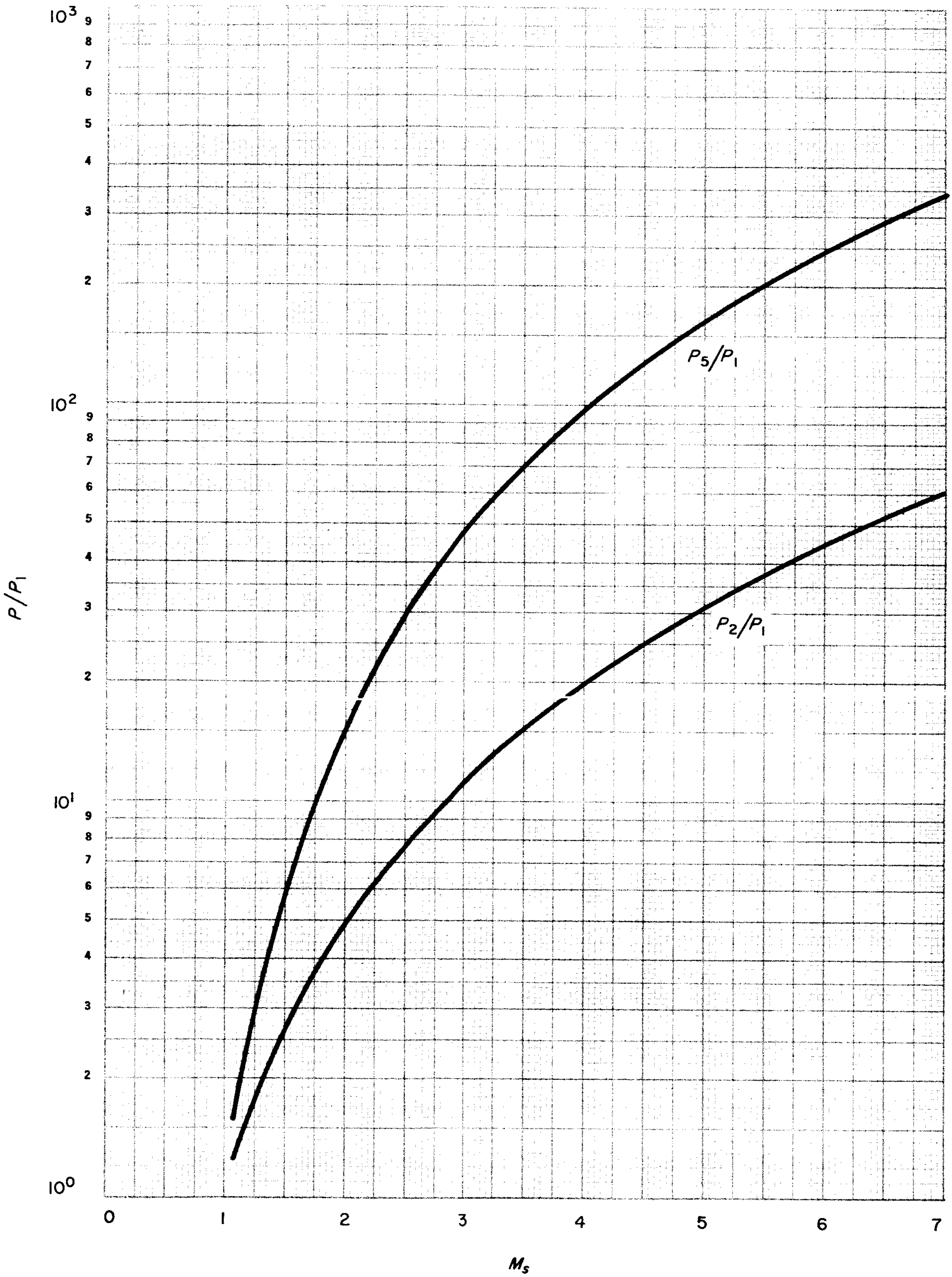

Fig. 1. Normal shock wave pressure ratio for an ideal gas ($\gamma = 5/3$)

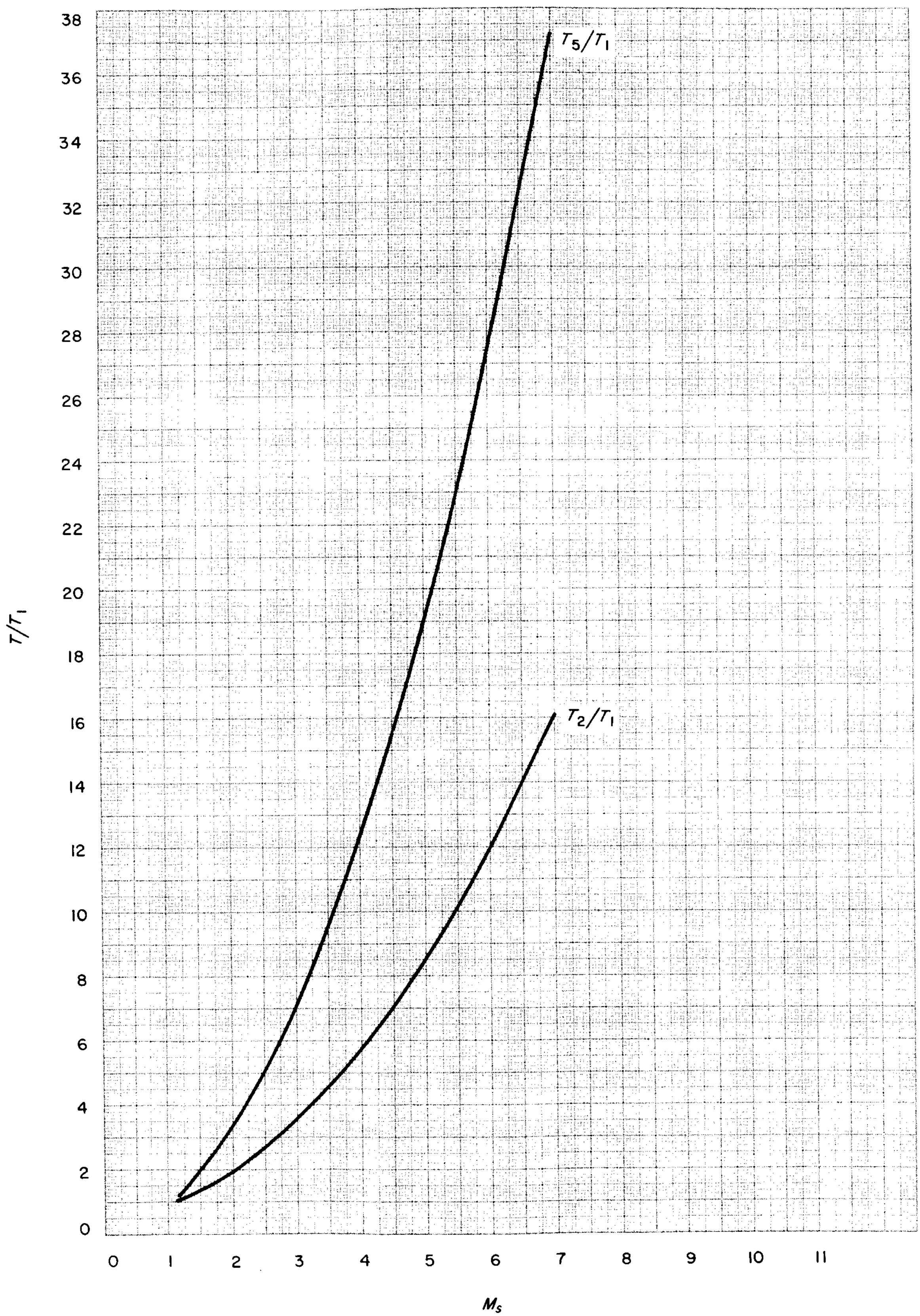

Fig. 2. Normal shock wave temperature ratio for an ideal gas ($\gamma = 5/3$)

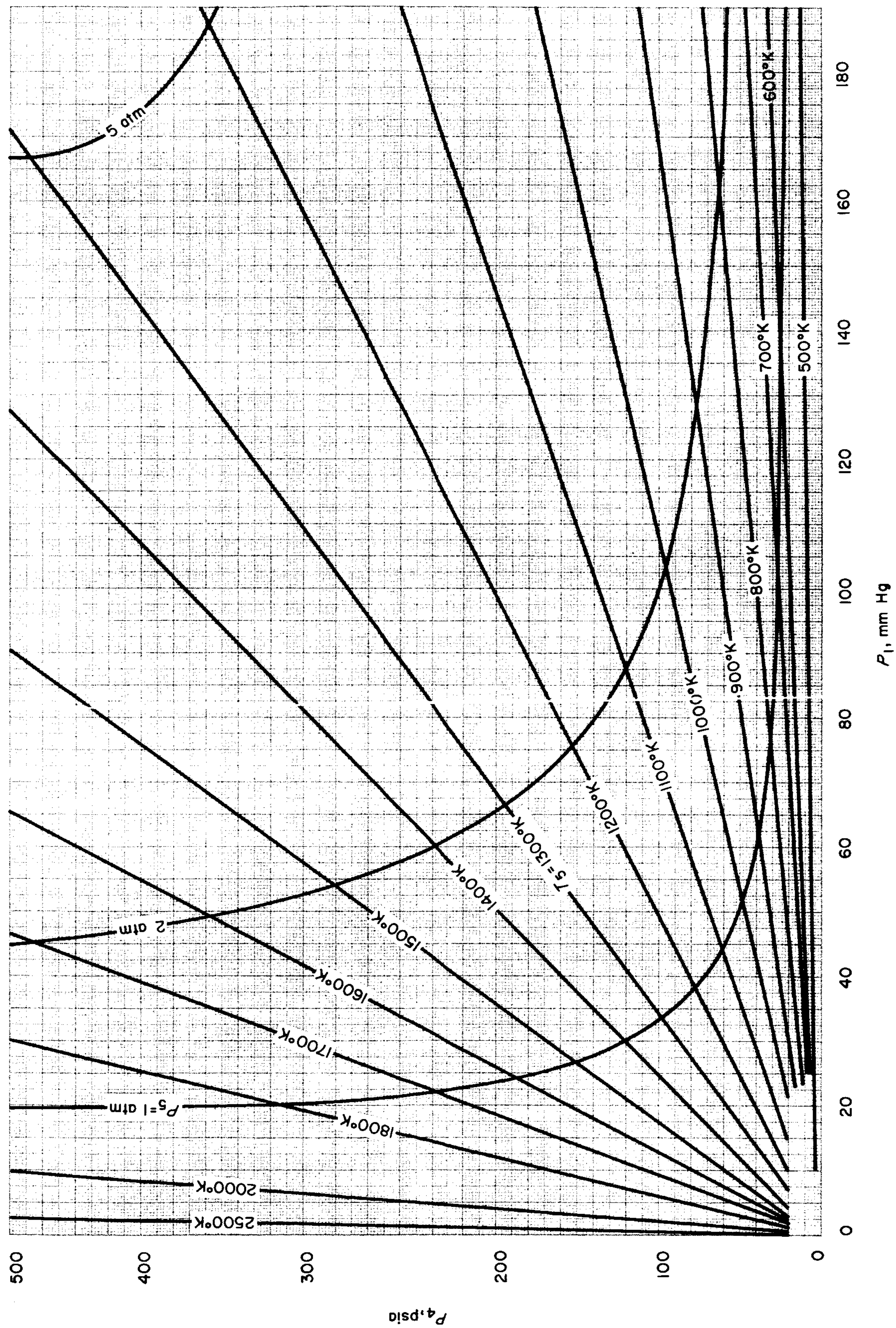

Fig. 3. Stagnation conditions obtainable with various initial shock tube pressures for helium driving helium

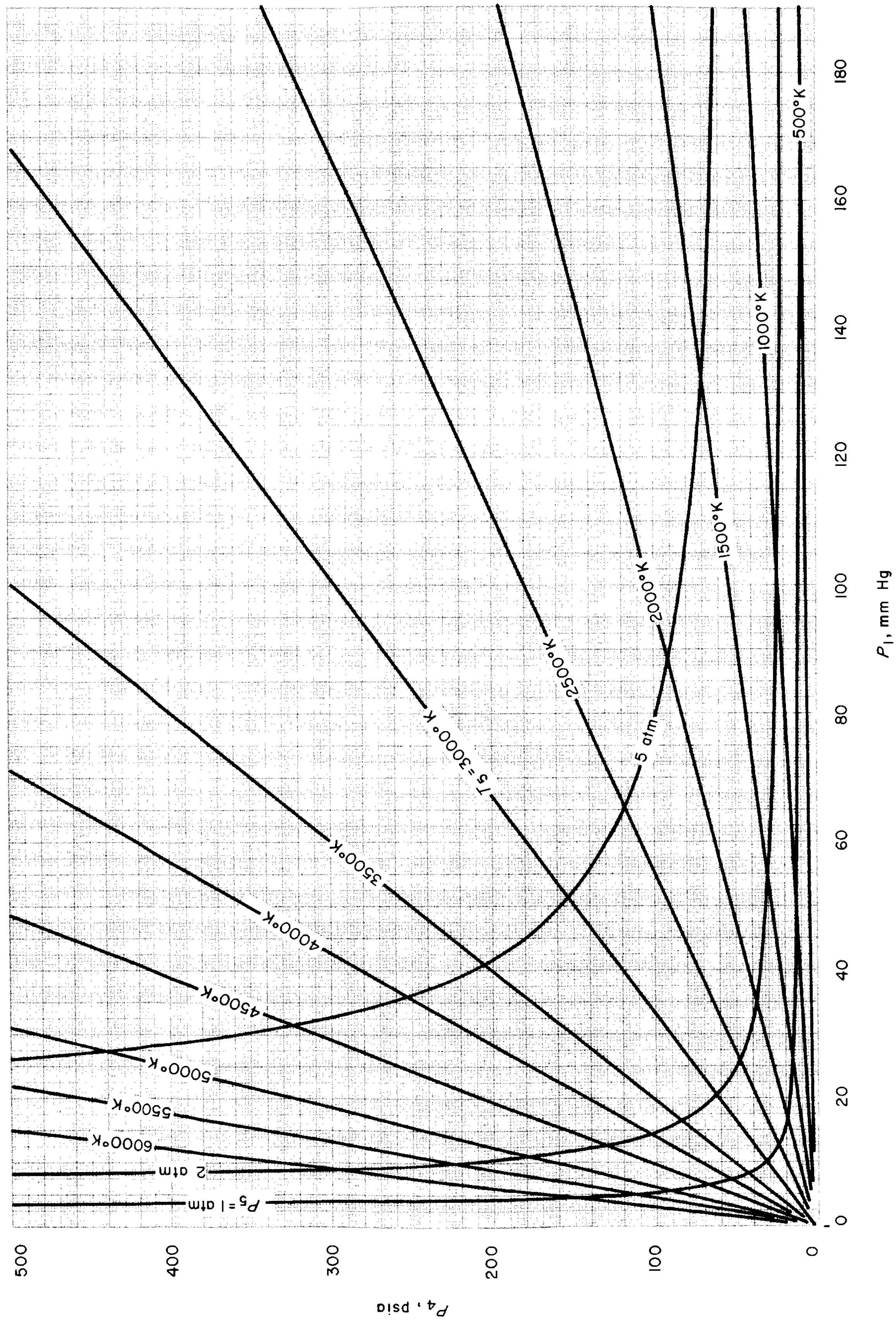

Fig. 4. Stagnation conditions obtainable with various initial shock tube pressures for helium driving neon

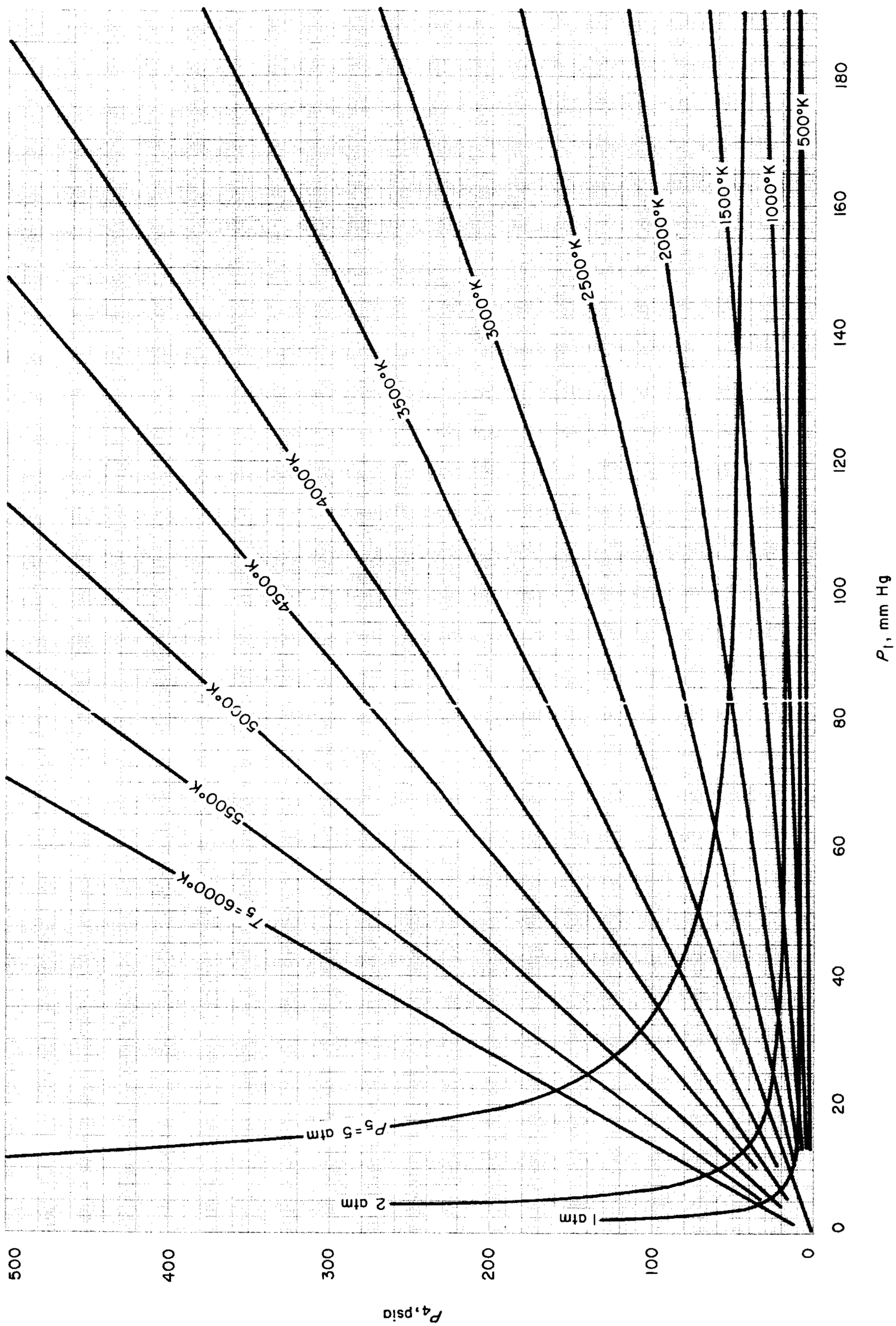

Fig. 5. Stagnation conditions obtainable with various initial shock tube pressures for helium driving argon

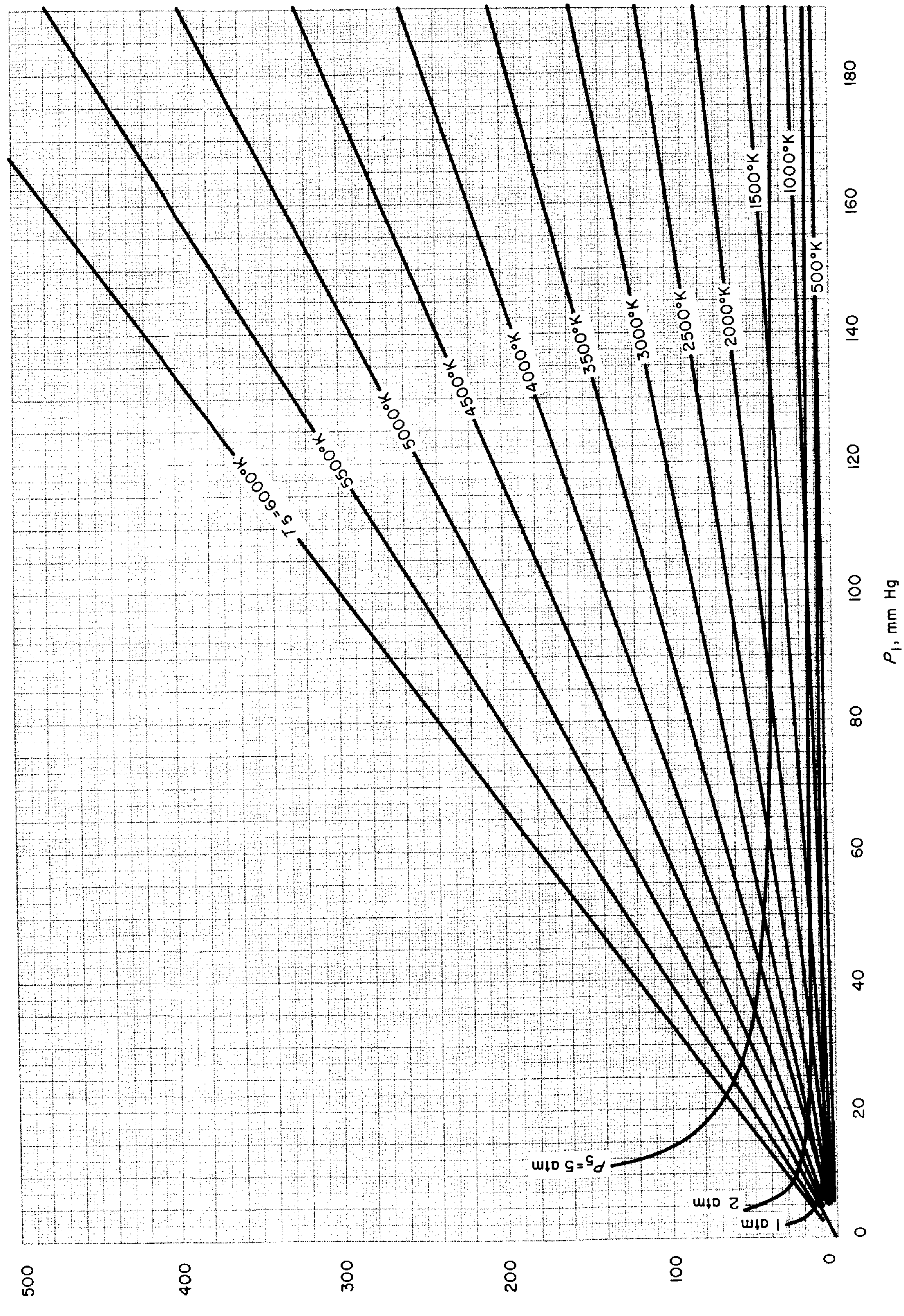

Fig. 6. Stagnation conditions obtainable with various initial shock tube pressures for helium driving krypton

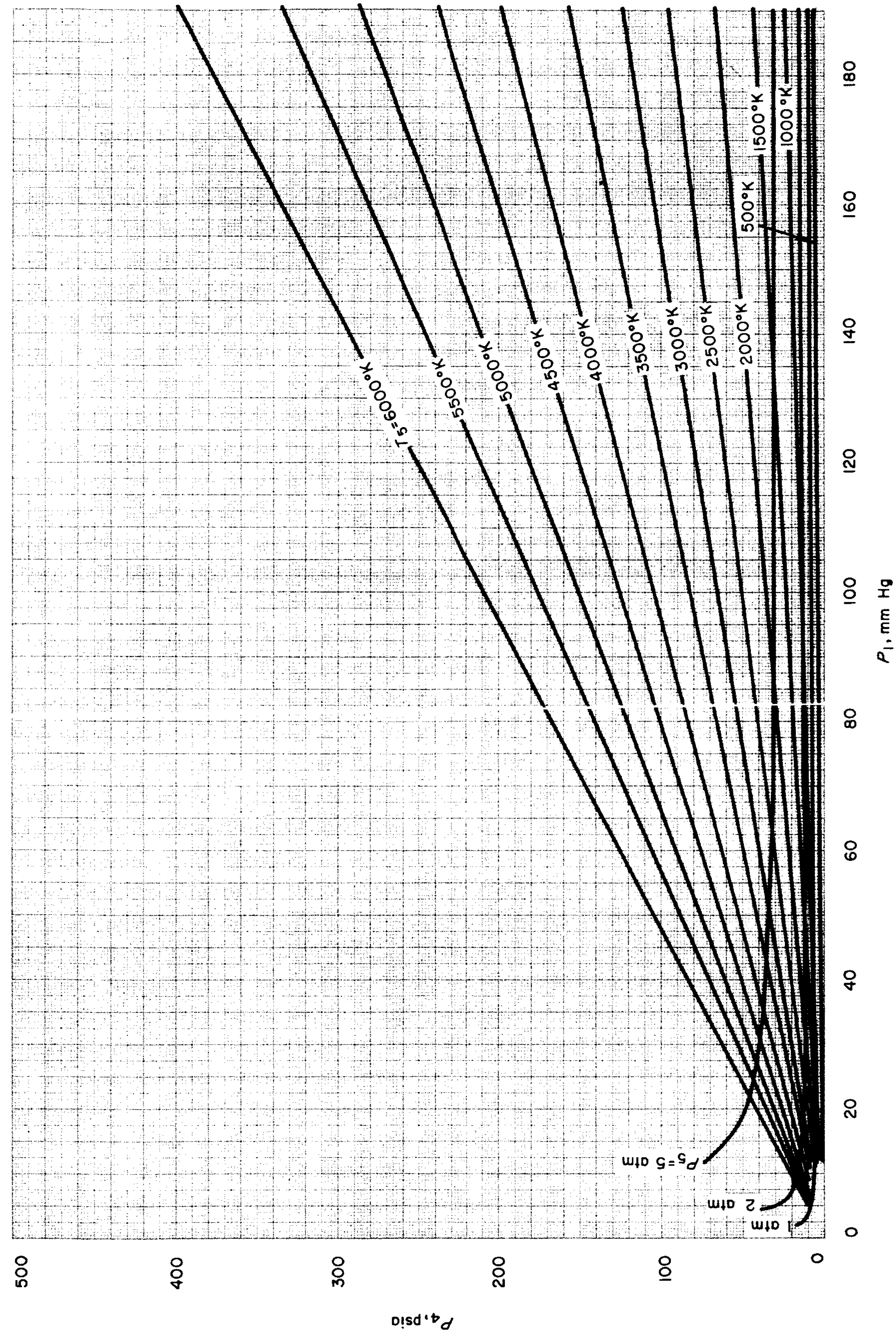

Fig. 7. Stagnation conditions obtainable with various initial shock tube pressures for helium driving xenon

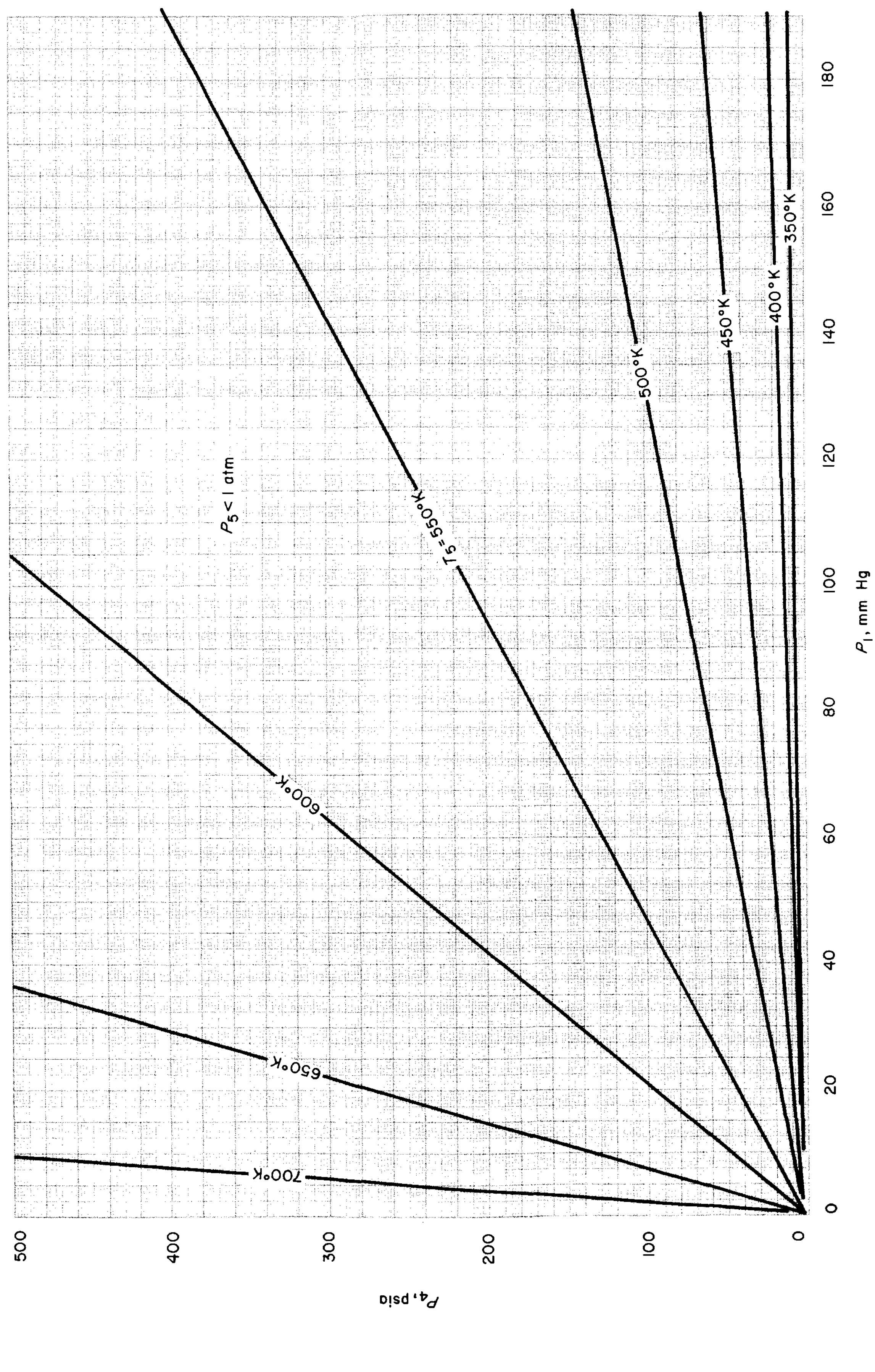

Fig. 8. Stagnation conditions obtainable with various initial shock tube pressures for argon driving helium

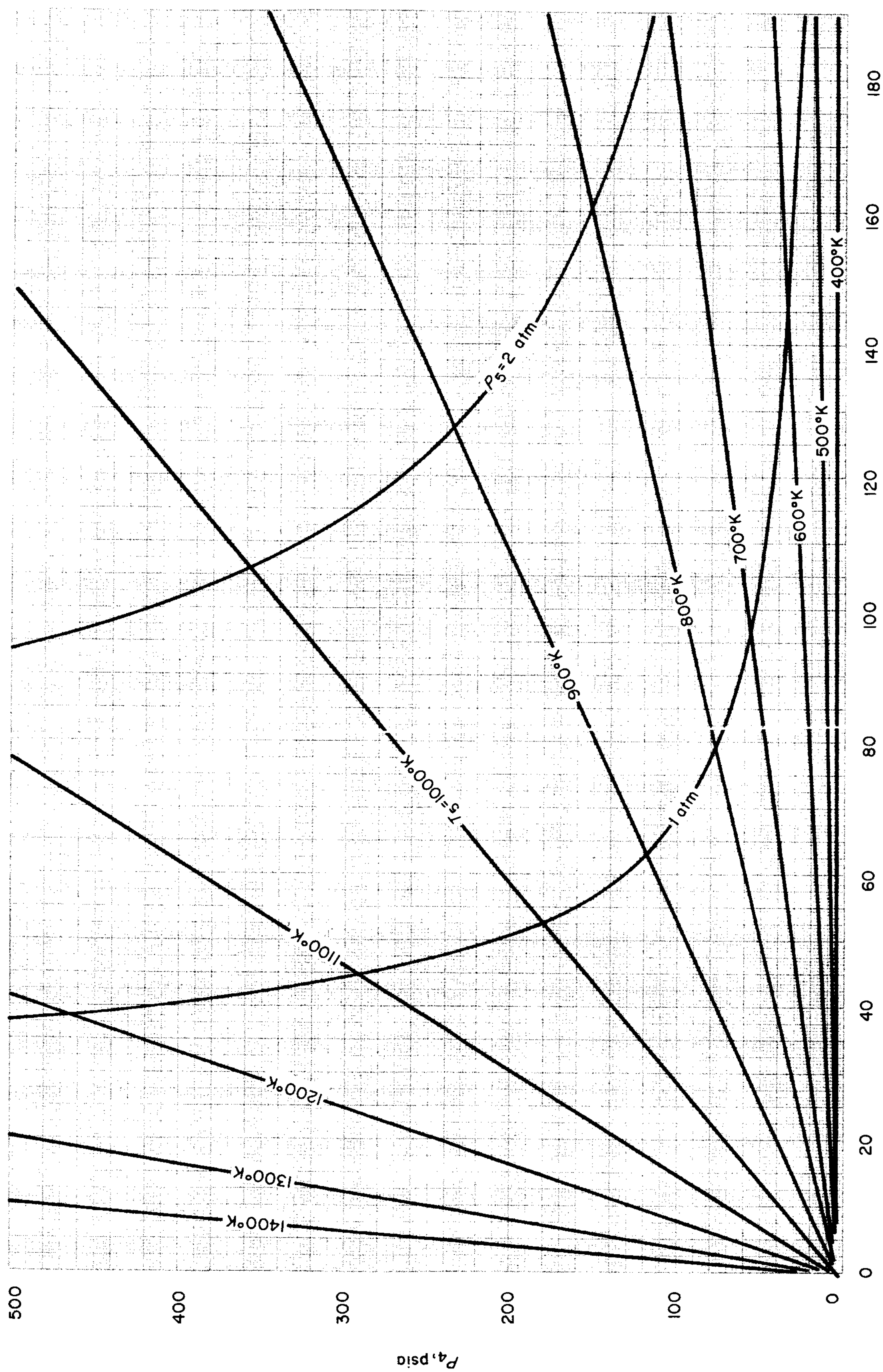

Fig. 9. Stagnation conditions obtainable with various initial shock tube pressures for argon driving neon

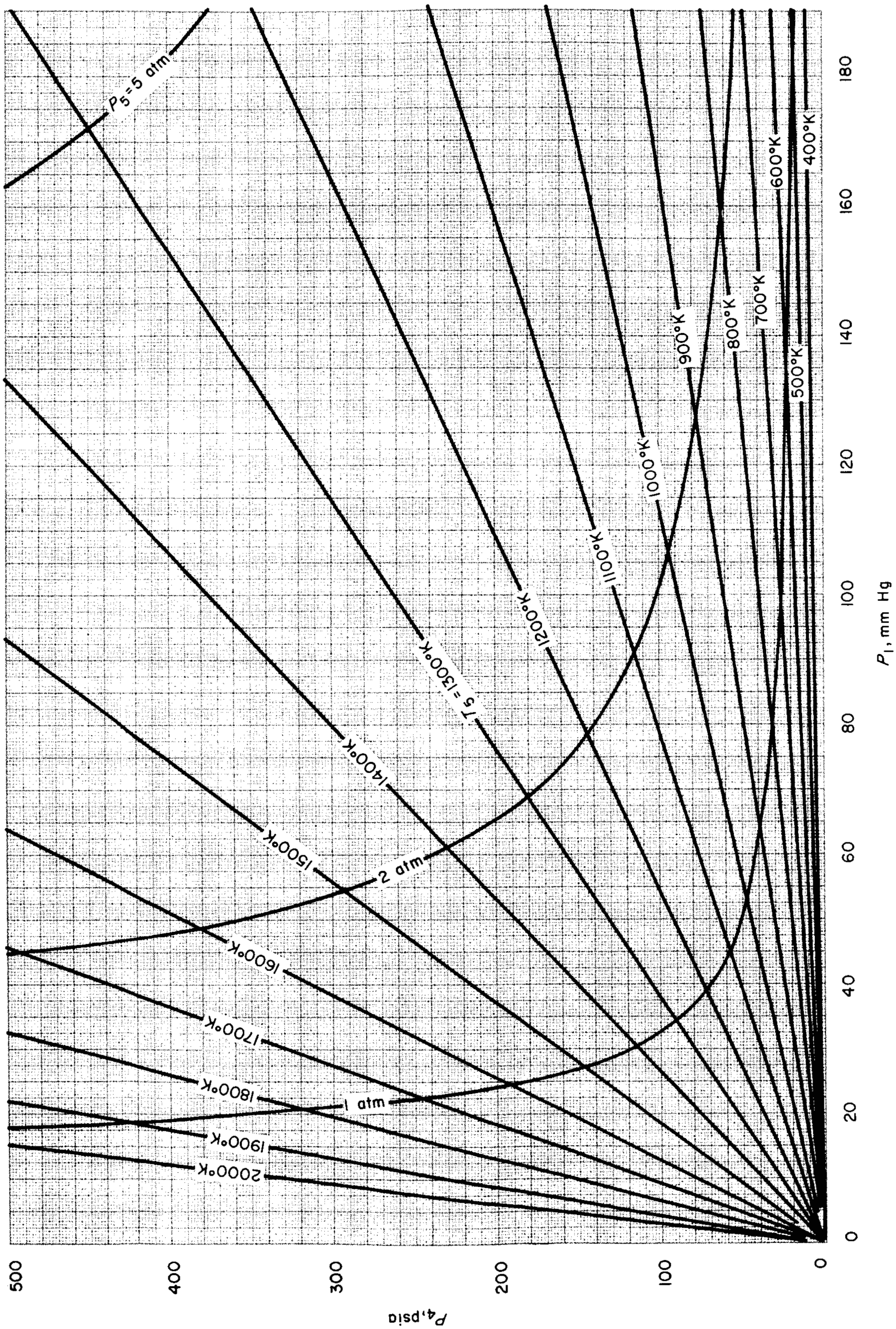

Fig. 10. Stagnation conditions obtainable with various initial shock tube pressures for argon driving argon

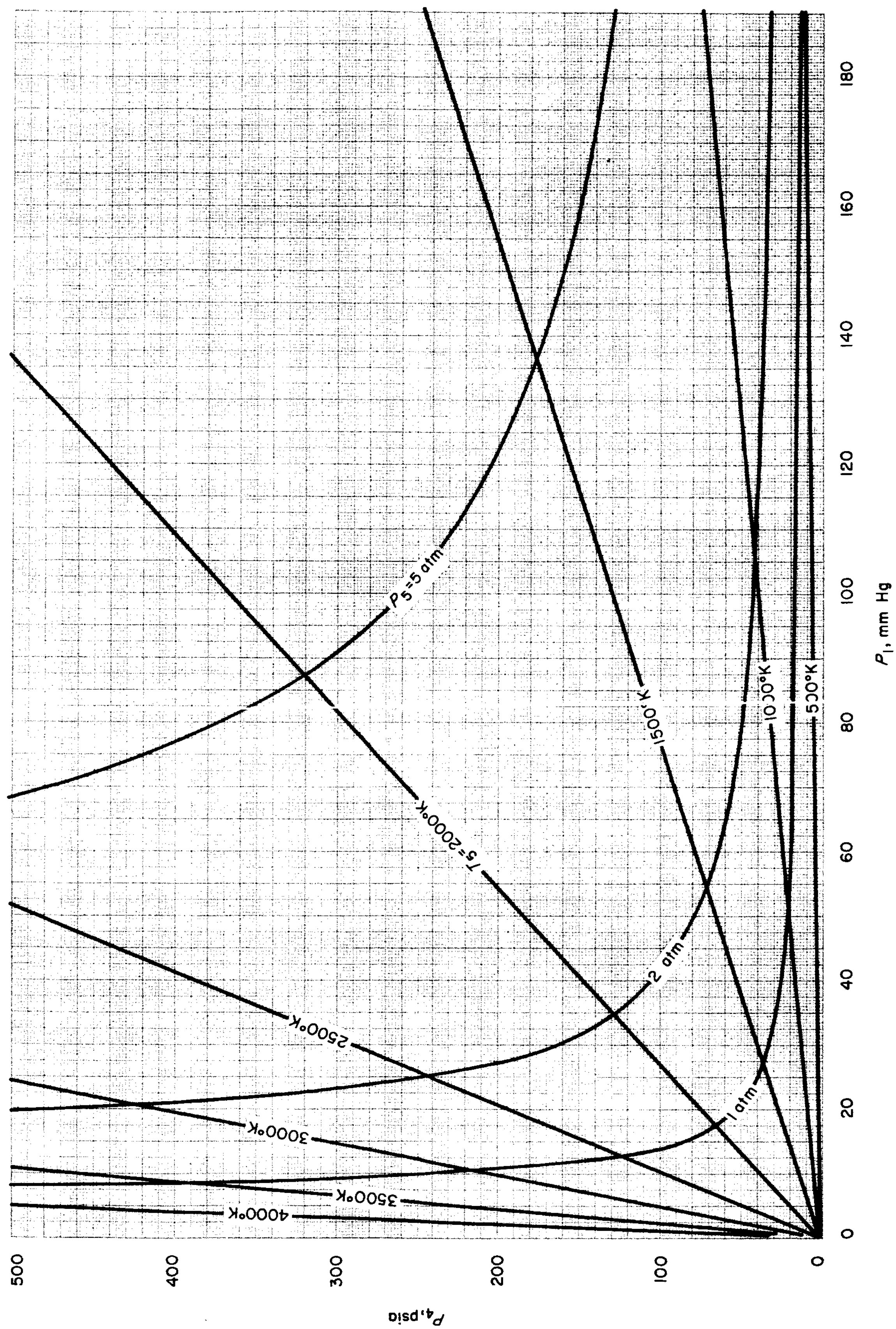

Fig. 11. Stagnation conditions obtainable with various initial shock tube pressures for argon driving krypton

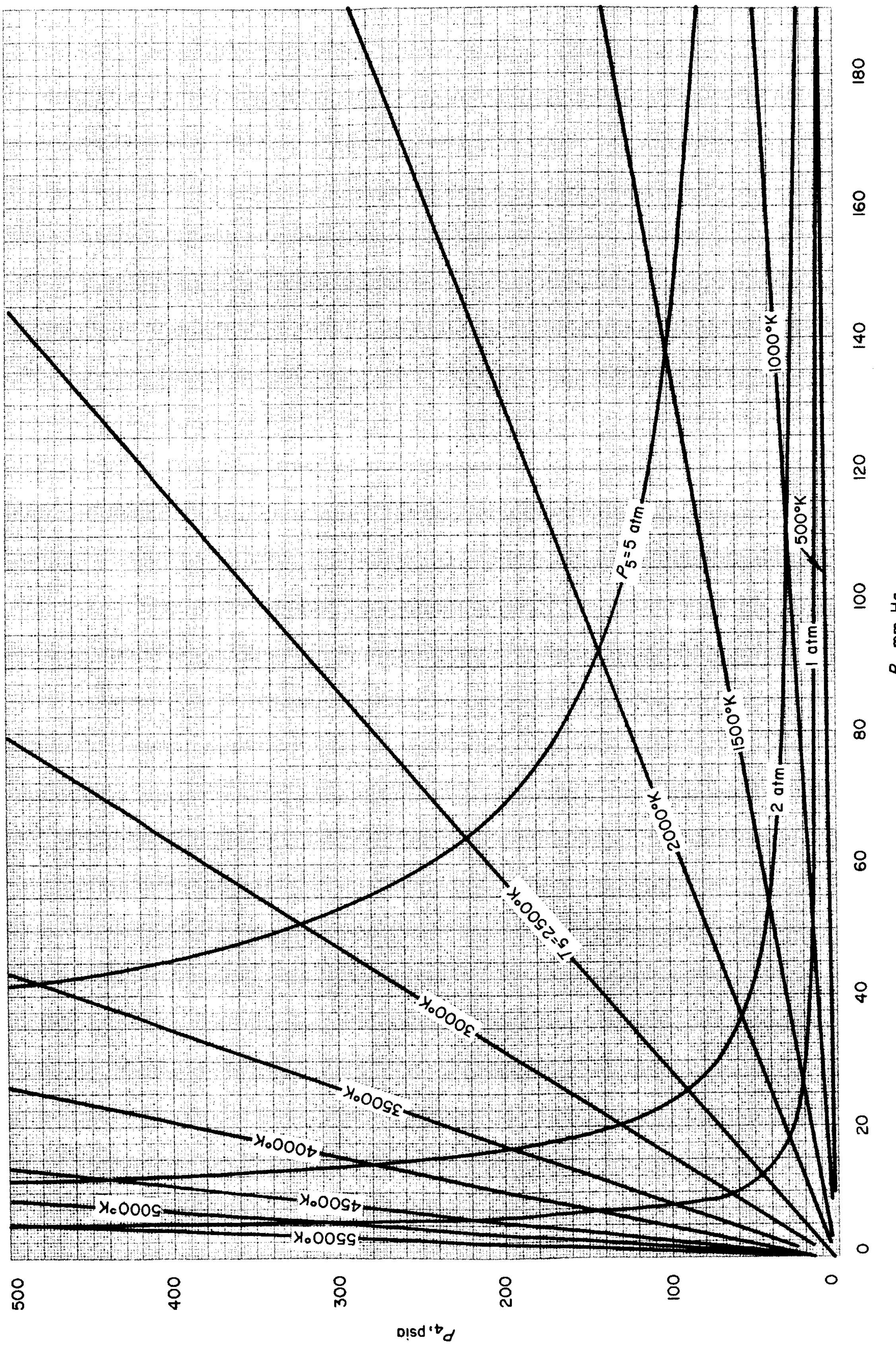

Fig. 12. Stagnation conditions obtainable with various initial shock tube pressures for argon driving xenon